

# Rapid Formation of Icy Super-Earths and the Cores of Gas Giant Planets

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## ABSTRACT

We describe a coagulation model that leads to the rapid formation of super-Earths and the cores of gas giant planets. Interaction of collision fragments with the gaseous disk is the crucial element of this model. The gas entrains small collision fragments, which rapidly settle to the disk midplane. Protoplanets accrete the fragments and grow to masses  $\gtrsim 1 M_{\oplus}$  in  $\sim 1$  Myr. Our model explains the mass distribution of planets in the Solar System and predicts that super-Earths form more frequently than gas giants in low mass disks.

*Subject headings:* planetary systems – solar system: formation – planets and satellites: formation

## 1. INTRODUCTION

Collisional cascades play a central role in planet formation. In current theory, planets grow from collisions and mergers of km-sized planetesimals in a gaseous disk. As planets grow, they stir leftover planetesimals along their orbits to high velocities. Eventually, collisions among planetesimals produce smaller fragments instead of larger, merged objects. Continued stirring leads to a cascade of destructive collisions which grinds the leftovers to dust. This process (i) explains the masses of terrestrial planets (Kenyon & Bromley 2006) and Kuiper belt objects (Kenyon et al. 2008) and (ii) produces debris disks similar to those observed around nearby main sequence stars (Wyatt 2008).

Numerical simulations of icy planet formation suggest the cascade limits the masses of growing protoplanets to  $\sim 0.01 M_{\oplus}$  (Kenyon & Bromley 2008, hereafter KB08). This mass is much smaller than the core mass,  $\gtrsim 0.1\text{--}1 M_{\oplus}$ , required for a protoplanet to accrete gas and become a gas giant planet (Pollack et al. 1996; Inaba et al. 2003; Alibert et al. 2005). Unless icy protoplanets can accrete collision fragments before the fragments are ground to dust, these protoplanets cannot grow into gas giant planet cores. Thus, finding a mechanism to halt the cascade is essential to form gas giant planets.

Here, we describe how interactions between the fragments and the gaseous disk can halt the cascade. In our picture, the gas traps small fragments with sizes of 0.1 mm to 1 m and prevents them from colliding at large velocities. These fragments then settle rapidly to the disk midplane, where protoplanets can accrete them. For a broad range of initial conditions, analytic results and detailed numerical simulations demonstrate that this process yields 1–10  $M_{\oplus}$  cores in 1–2 Myr.

We develop the analytic theory in §2 and derive the conditions needed for protoplanets to accrete collision fragments and grow to masses of  $\sim 1 M_{\oplus}$  in 1–2 Myr. We confirm these estimates in §3 with detailed numerical calculations. We conclude with a brief discussion in §4.

## 2. PHYSICAL MODEL

The crucial element of our model is the interaction of collision fragments with the gaseous disk. Fragments larger than the ‘stopping radius’  $r_s \approx 0.5\text{--}2$  m at 5–10 AU (Weidenschilling 1977; Rafikov 2004), orbit with the growing protoplanets, independently of the gas. Destructive collisions among these fragments fuel the collisional cascade. However, the gas entrains particles with radii  $r \lesssim r_s$ . These fragments orbit with the gas; thus, their velocity dispersions are small and independent of massive protoplanets. By trapping small collision fragments, the gas halts the collisional cascade.

The gas also allows protoplanets to accrete the debris. When the collisional cascade begins, the mass in leftover planetesimals is  $\sim 1\text{--}10 M_{\oplus}$ . The cascade grinds all of this mass into small fragments which are trapped by the gas. Most of the trapped fragments fall through the gas into the midplane of the disk, where growing protoplanets accrete them. Protoplanets that accrete  $\sim 0.1\text{--}1 M_{\oplus}$  before the gas dissipates ( $\sim 3\text{--}10$  Myr; Hartmann et al. 1998; Haisch, Lada, & Lada 2001; Kennedy & Kenyon 2009) become gas giants. Thus, our model yields gas giant cores if (i) the collisional cascade produces fragments fast enough, (ii) the fragments quickly settle to the midplane, and (iii) the largest protoplanets rapidly

accrete the fragments.

To examine whether this physical model leads to cores with masses of  $\sim 1 M_{\oplus}$ , we consider the growth of planets in a disk of gas and icy objects around a star with mass  $M_{\star}$ . Material at a distance  $a$  from the central star orbits with angular frequency  $\Omega$  and has surface densities  $\Sigma_s$  (solids) and  $\Sigma_g$  (gas). We adopt a solid-to-gas ratio of 1:100 and  $\Sigma_s = \Sigma_{s,0} x_m a^{-3/2}$ , where  $\Sigma_{s,0} = 2.5 \text{ g cm}^{-2}$  at 5 AU and  $x_m$  is a scale factor.

Forming icy protoplanets is the first step in our model. In an ensemble of 1 km planetesimals, collisional growth yields a few 1000 km objects – ‘oligarchs’ – that contain an ever-increasing fraction of the mass in solids (Ida & Makino 1993; Wetherill & Stewart 1993; Rafikov 2003). From numerical simulations of planet growth at 30–150 AU, the timescale to produce an oligarch around a solar-type star is (KB08)

$$t_{1000} \sim 10^5 x_m^{-1.15} \left( \frac{a}{5 \text{ AU}} \right)^3 \text{ yr} . \quad (1)$$

Thus, oligarchs form at 5 AU before the gas dissipates.

Once oligarchs form, collisions among leftover planetesimals produce copious amounts of fragments. In the high velocity limit, the collision time for a planetesimal of mass  $M$  in a swarm of icy planetesimals with mass  $M$ , radius  $r$ , and surface density  $\Sigma$  is  $t_c \approx M/(\Sigma \pi r^2 \Omega)$  (Goldreich, Lithwick, & Sari 2004). Thus,

$$t_c \approx 10^5 x_m^{-1} \left( \frac{r}{1 \text{ km}} \right) \left( \frac{a}{5 \text{ AU}} \right)^{3/2} \text{ yr} . \quad (2)$$

Collisions among planetesimals produce debris at a rate  $\dot{M} \approx N \delta M t_c^{-1}$ , where  $N$  is the number of planetesimals of mass  $M$  and  $\delta M$  is the mass in fragments produced in a single collision. In an annulus of width  $\delta a$  at distance  $a$  from the central star,  $N \approx 2\pi \Sigma a \delta a/M$ . If  $\sim 10\%$  of the mass in each collision is converted into fragments

$$\dot{M}_f \approx 4 \times 10^{-7} x_m^2 \left( \frac{5 \text{ AU}}{a} \right)^{7/2} \left( \frac{\delta a}{0.2 \text{ AU}} \right) \left( \frac{1 \text{ km}}{r} \right) M_{\oplus} \text{ yr}^{-1} , \quad (3)$$

where we have set the width of the annulus equal to the width of the ‘feeding zone’ for a  $0.1 M_{\oplus}$  protoplanet (Lissauer 1987). Thus, disks with  $x_m \gtrsim 1\text{--}2$  produce fragments at a rate sufficient to form  $\gtrsim 1 M_{\oplus}$  cores in 1–2 Myr.

Most of the mass in fragments settles quickly to the disk midplane. For a settling time  $t_s \approx 4x_m(1 \text{ m}/r) \text{ yr}$  (Chiang & Goldreich 1997), fragments with  $r \gtrsim 0.1 \text{ mm}$  reach the midplane on the collision timescale of  $\sim 10^5 \text{ yr}$  (Eq. 2). For a size distribution  $n(m) \propto m^q$  with  $q = -1$  to  $-0.8$  (Dohnanyi 1969; Holsapple & Housen 2007), 66% to 99% of the total mass in fragments with  $r \lesssim 1 \text{ m}$  settles to the midplane in  $\lesssim 10^5 \text{ yr}$  at 5–10 AU.

Oligarchs rapidly accrete fragments in the midplane. The maximum accretion rate for an oligarch with  $M_o \sim 0.01 M_\oplus$  at 5 AU is  $\sim 5 \times 10^{-6} M_\oplus \text{ yr}^{-1}$  (Rafikov 2004). This maximum rate yields  $5 M_\oplus$  cores in 1 Myr. At the onset of the cascade, our simulations suggests oligarchs at 5 AU accrete at a rate

$$\dot{M}_o \approx 10^{-6} \left( \frac{M_o}{0.01 M_\oplus} \right)^{2/3} \left( \frac{M_f}{1 M_\oplus} \right) M_\oplus \text{ yr}^{-1}, \quad (4)$$

where  $M_f$  is the total mass in fragments in a feeding zone with width  $\delta a \approx 0.2 \text{ AU}$  at 5 AU. Thus, protoplanets likely reach masses  $\sim 1 M_\oplus$  in 1–2 Myr.

These analytic estimates confirm the basic aspects of our model. In a gaseous disk with  $\Sigma_g \approx 250 \text{ g cm}^{-2}$ , the gas halts the collisional cascade. Collision fragments entrained by the gas rapidly settle to the midplane. Protoplanets with masses  $\sim 0.01 M_\oplus$  can accrete collision fragments rapidly and grow to masses  $\sim 1 M_\oplus$  before the gas dissipates in 3–10 Myr.

### 3. NUMERICAL MODEL

To explore this picture in more detail, we calculate the formation of cores with our hybrid multiannulus coagulation– $n$ -body code (Bromley & Kenyon 2006). In previous calculations, we followed the evolution of objects with  $r \gtrsim r_s$ ; collision fragments with  $r \lesssim r_s$  were removed by the collisional cascade (KB08). Here, we include the evolution of small fragments entrained by the gas. We follow Brauer et al. (2008a) and calculate the scale height of small particles with  $r < r_s$  as  $H = \alpha H_g / [\min(St, 0.5)(1 + St)]$ , where  $H_g$  is the scale height of the gas (Kenyon & Hartmann 1987, KB08),  $\alpha$  is the turbulent viscosity of the gas, and  $St = r\rho_s\Omega/c_s\rho_g$ . In this expression for the Stokes number ( $St$ ),  $c_s$  is the sound speed of the gas,  $\rho_g$  is the gas density, and  $\rho_s$  is the mass density of a fragment. We assume small particles have vertical velocity  $v = H\Omega$  and horizontal velocity  $h = 1.6v$ . Protoplanets accrete fragments at a rate  $n\sigma v_{rel}$ , where  $n$  is the number density of fragments,  $\sigma$  is the cross-section (including gravitational focusing), and  $v_{rel}$  is the relative velocity (e.g., Kenyon & Luu 1998, Appendix A.2). Although this approximation neglects many details of the motion of particles in the gas (Brauer et al. 2008a), it approximates the dynamics and structure of the fragments reasonably well and allows us to calculate accretion of fragments by much larger oligarchs.

Using the statistical approach of Safronov (1969), we evolve the masses and orbits of planetesimals in a set of concentric annuli with widths  $\delta a_i$  at distances  $a_i$  from the central star (KB08). The calculations use realistic cross-sections (including gravitational focusing) to derive collision rates (Spaute et al. 1991) and a Fokker-Planck algorithm to derive gravitational stirring rates (Ohtsuki, Stewart, & Ida 2002). When large objects reach a mass

$M_{pro}$ , we ‘promote’ them into an  $n$ -body code (Bromley & Kenyon 2006). This code follows the trajectories of individual objects and includes algorithms to allow interactions between the massive  $n$ -bodies and less massive objects in the coagulation code.

To assign collision outcomes, we use the ratio of the center of mass collision energy  $Q_c$  and the energy needed to eject half the mass of a pair of colliding planetesimals to infinity  $Q_d^*$ . We adopt  $Q_d^* = Q_b r^{\beta_b} + Q_g \rho_g r^{\beta_g}$  (Benz & Asphaug 1999), where  $Q_b r^{\beta_b}$  is the bulk component of the binding energy,  $Q_g \rho_g r^{\beta_g}$  is the gravity component of the binding energy, and  $\rho_g$  is the mass density of a planetesimal. The mass of a merged pair is  $M_1 + M_2 - M_{ej}$ , where the mass ejected in the collision is  $M_{ej} = 0.5(M_1 + M_2)(Q_c/Q_d^*)^{9/8}$  (Kenyon & Luu 1999).

Consistent with recent  $n$ -body simulations, we consider two sets of fragmentation parameters  $f_i$ . Strong planetesimals have  $f_S = (Q_b = 1, 10^3, \text{ or } 10^5 \text{ erg g}^{-1}, \beta_b \approx 0, Q_g = 1.5 \text{ erg g}^{-1} \text{ cm}^{-1.25}, \beta_g = 1.25; \text{ KB08, Benz \& Asphaug 1999})$ . Weaker planetesimals have  $f_W = (Q_b = 2 \times 10^5 \text{ erg g}^{-1} \text{ cm}^{0.4}, \beta_b \approx -0.4, Q_g = 0.22 \text{ erg g}^{-1} \text{ cm}^{-1.3}, \beta_g = 1.3; \text{ Leinhardt \& Stewart 2008})$ .

Our initial conditions are appropriate for a disk around a young star (e.g. Dullemond & Dominik 2005; Ciesla 2007a; Garaud 2007; Brauer et al. 2008b). We consider systems of 32 annuli with  $a_i = 5\text{--}10 \text{ AU}$  and  $\delta a_i/a_i = 0.025$ . The disk is composed of small planetesimals with radii ranging from  $r_{min} = r_s \approx 0.5\text{--}2 \text{ m}$  (Rafikov 2004) to  $r_0 = 1 \text{ km}, 10 \text{ km}, \text{ or } 100 \text{ km}$  and an initial mass distribution  $n_i(M_{ik}) \propto M_{ik}^{-0.17}$ . The mass ratio between adjacent bins is  $\delta = M_{ik+1}/M_{ik} = 1.4\text{--}2$  (e.g., Kenyon & Luu 1998, KB08). Each bin has the same initial eccentricity  $e_0 = 10^{-4}$  and inclination  $i_0 = e_0/2$ .

For each combination of  $r_0$ ,  $f_i$ , and  $x_m = 1\text{--}5$ , we calculate the growth of oligarchs with two different approaches to grain accretion. In models without grain accretion, fragments with  $r \lesssim r_{min}$  are ‘lost’ to the grid. Oligarchs cannot accrete these fragments; their masses stall at  $M \lesssim 0.1 \text{ M}_\oplus$ . In models with grain accretion, we track the abundances of fragments with  $0.1 \text{ mm} \lesssim r \lesssim r_{min}$  which settle to the disk midplane on short timescales. Oligarchs can accrete these fragments; they grow rapidly at rates set by the production of collision fragments.

For the gaseous disk, we adopt  $\alpha = 10^{-4}$ , an initial surface density,  $\Sigma_{g,0} = 100 \Sigma_{s,0} x_m a^{-3/2}$ , and a depletion time  $t_g = 3 \text{ Myr}$ . The surface density at later times is  $\Sigma_{g,t} = \Sigma_{g,0} e^{-t/t_g}$ . We ignore the migration of protoplanets from torques between the gas and the planet (Lin & Papaloizou 1986; Ward 1997). Alibert et al. (2005) show that migration enhances growth of protoplanets; thus our approach underestimates the growth time. We also ignore the radial drift of fragments coupled to the gas. Depending on the internal structure of

the disk, fragments can drift inward, drift outward, or become concentrated within local pressure maxima or turbulent eddies (Weidenschilling 1977; Haghighipour & Boss 2003; Inaba & Barge 2006; Masset et al. 2006; Ciesla 2007b; Kretke & Lin 2007; Kato et al. 2008). Here, our goal is to provide a reasonable first estimate for the growth rates of protoplanets. We plan to explore the consequences of radial drift in subsequent papers.

#### 4. RESULTS

Fig. 1 shows mass histograms at 1–10 Myr for coagulation calculations without grain accretion using  $r_0 = 1$  km and the strong fragmentation parameters ( $f_S$ ). After the first oligarchs with  $M \sim 0.01 M_\oplus$  form at  $\sim 0.1$  Myr, the collisional cascade starts to remove leftover planetesimals from the grid. Independent of  $Q_b$ , the cascade removes  $\sim 50\%$  of the initial mass of the grid in  $\sim 4 x_m^{-1.25}$  Myr. As the cascade proceeds, growth of the largest oligarchs stalls at a maximum mass  $M_{o,max} \approx 0.1 x_m M_\oplus$ .

These results depend weakly on  $r_0$ . The time to produce the first oligarch with  $r \sim 1000$  km increases with  $r_0$ ,  $t_{1000} \sim 0.1 x_m^{-1.25} (r_0/10 \text{ km})^{1/2}$  Myr. Calculations with larger  $r_0$  tend to produce larger oligarchs at 10 Myr:  $M_{o,max} \approx 1 M_\oplus$  ( $2 M_\oplus$ ) for  $r_0 = 10$  km ( $100$  km). In  $\sim 50$  calculations, none produce cores with  $M_{o,max} \gtrsim 1 M_\oplus$  on timescales of  $\lesssim 10$  Myr.

For  $r_0 \lesssim 100$  km, our results depend on  $f_i$ . In models with  $r_0 = 1$  km and  $10$  km, the  $f_W$  fragmentation parameters yield oligarchs with smaller maximum masses,  $M_{o,max} \approx 0.3$ – $0.6 M_\oplus$ . Because leftover planetesimals with  $r \sim 1$ – $10$  km fragment more easily, the cascade begins (and growth stalls) at smaller collision velocities when oligarchs are less massive (Kenyon et al. 2008).

Calculations with grain accretion produce cores rapidly. Fig. 2 shows results at 1–10 Myr for calculations with  $r_0 = 1$  km and the  $f_S$  fragmentation parameters. As the first oligarchs reach masses of  $\sim 0.01 M_\oplus$  at 0.1 Myr, the cascade generates many small collision fragments with  $r \sim 1$  mm to 1 m. These fragments rapidly settle to the disk midplane and grow to sizes of 0.1–1 m. When the cascade has shattered  $\sim 25\%$  of the leftover planetesimals, oligarchs begin a second phase of runaway growth by rapidly accreting small particles in the midplane. For calculations with  $x_m = 1$ – $5$ , it takes  $\sim 1$ – $2 x_m^{-1.25}$  Myr to produce at least one core with  $M_o \gtrsim 1$ – $5 M_\oplus$ . Thus, cores form before the gas dissipates.

These results depend on  $r_0$ . For  $r_0 = 10$  km, fragmentation produces small grains 2–3 times more slowly than calculations with  $r_0 = 1$  km. These models form cores more slowly, in  $5$ – $10 x_m^{-1.25}$  Myr instead of  $1$ – $2 x_m^{-1.25}$  Myr. For models with  $r_0 = 100$  km, fragmentation yields a negligible mass in small grains. Thus, cores never form in  $\lesssim 10$ – $20$  Myr.

The timescales to form cores also depend on  $f_i$ . Calculations with the  $f_W$  parameters form cores 10% to 20% faster than models with the  $f_S$  parameters.

## 5. CONCLUSIONS

Gaseous disks are a crucial element in the formation of the cores of gas giant planets. The gas traps small collision fragments and halts the collisional cascade. Once fragments settle to the disk midplane, oligarchs accrete the fragments and grow to masses  $\gtrsim 1 M_\oplus$  in 1–3 Myr.

Our model predicts two outcomes for icy planet formation. Oligarchs that form before (after) the gas disk dissipates reach maximum masses  $\gtrsim 1 M_\oplus$  ( $\lesssim 0.01$ – $0.1 M_\oplus$ ). Setting the timescale to form a 1000 km oligarch (Eq. 1) equal to the gas dissipation timescale  $t_g$  yields a boundary between these two types of icy protoplanets at  $a_g \approx 15 x_m^{0.4} (t_g/3 \text{ Myr})^{1/3}$  AU. We expect massive cores at  $a \lesssim a_g$  and low mass icy protoplanets at  $a \gtrsim a_g$ .

This prediction has a clear application to the Solar System. Recent dynamical calculations suggest that the Solar System formed with four gas giants at 5–15 AU and an ensemble of Pluto-mass and smaller objects beyond 20 AU (Morbidelli et al. 2008). For a protosolar disk with  $x_m^{1.2} (t_g/3 \text{ Myr}) \approx 1$ , our model explains this configuration. Disks with these parameters are also common in nearby star-forming regions (Andrews & Williams 2005). Thus, our results imply planetary systems like our own are common.

Our model yields a large range of final masses for massive icy cores. Protoplanets that grow to a few  $M_\oplus$  well before the gas dissipates can accrete large amounts of gas from the disk and become gas giants (Pollack et al. 1996; Alibert et al. 2005). Protoplanets that grow more slowly cannot accrete much gas and become icy ‘super-Earths’ with much lower masses (Kennedy et al. 2006; Kennedy & Kenyon 2008). For solar-type stars with  $t_g \approx 3$  Myr, our results suggest that gas giants (super-Earths) are more likely in disks with  $x_m \gtrsim 1.5$  ( $x_m \lesssim 1.5$ ) at 5–10 AU.

Testing this prediction requires (i) extending our theory to a range of stellar masses and (ii) more detections of massive planets around lower mass stars. We plan to explore the consequences of our model for other stellar masses in future papers. Larger samples of planetary systems will test the apparent trend that gas giants (super-Earths) are much more common around solar-type (lower mass) stars (e.g., Cumming et al. 2008; Forveille et al. 2008). Comparing the results of our planned numerical calculations with these additional observations will yield a clear test of our model.

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## REFERENCES

Alibert, Y., Mordasini, C., Benz, W., & Winisdoerffer, C. 2005, A&A, 434, 343

Andrews, S. M., & Williams, J. P. 2005, ApJ, 631, 1134

Benz, W., & Asphaug, E. 1999, Icarus, 142, 5

Brauer, F., Dullemond, C. P., & Henning, T. 2008a, A&A, 480, 859

Brauer, F., Henning, T., & Dullemond, C. P. 2008, A&A, 487, L1

Bromley, B., & Kenyon, S. J. 2006, AJ, 131, 2737

Chiang, E. I., & Goldreich, P. 1997, ApJ, 490, 368

Ciesla, F. J. 2007, ApJ, 654, L159

Ciesla, F. J. 2007, Science, 318, 613

Cumming, A., Butler, R. P., Marcy, G. W., Vogt, S. S., Wright, J. T., & Fischer, D. A. 2008, PASP, 120, 531

Dohnanyi, J. W. 1969, J. Geophys. Res., 74, 2531

Dullemond, C. P., & Dominik, C. 2005, A&A, 434, 971

Forveille, T., et al. 2008, ArXiv e-prints, 809, arXiv:0809.0750

Garaud, P. 2007, ApJ, 671, 2091

Goldreich, P., Lithwick, Y., & Sari, R. 2004, ARA&A, 42, 549

Haghighipour, N., & Boss, A. P. 2003, ApJ, 598, 1301

Haisch, K., Lada, E. A., & Lada, C. J. 2001, ApJ, 553, 153

Hartmann, L., Calvet, N., Gullbring, E., & D'Alessio, P. 1998, *ApJ*, 495, 385

Holsapple, K. A., & Housen, K. R. 2007, *Icarus*, 191, 586

Ida, S., & Makino, J. 1993, *Icarus*, 106, 210

Inaba, S., & Barge, P. 2006, *ApJ*, 649, 415

Inaba, S., Wetherill, G. W., & Ikoma, M. 2003, *Icarus*, 166, 46

Kato, M. T., Nakamura, K., Tandokoro, R., Fujimoto, M., & Ida, S. 2008, arXiv:0810.3466

Kennedy, G. M., Kenyon, S. J., & Bromley, B. C. 2006, *ApJ*, 650, L139

Kennedy, G. M., & Kenyon, S. J. 2008, *ApJ*, 682, 1264

Kennedy, G. M., & Kenyon, S. J. 2009, *ApJ*, submitted

Kenyon, S. J., & Bromley, B. C. 2006, *AJ*, 131, 1837

Kenyon, S. J., & Bromley, B. C. 2008, *ApJS*, 179, No. 2

Kenyon, S. J., Bromley, B. C., O'Brien, D. P., & Davis, D. R. 2008, *The Solar System Beyond Neptune*, eds. M. A. Barucci, H. Boehnhardt, D. P. Cruikshank & A. Morbidelli, Tucson, University of Arizona Press, p. 293

Kenyon, S. J., & Hartmann, L. 1987, *ApJ*, 323, 714

Kenyon, S. J., & Luu, J. X. 1998, *AJ*, 115, 2136

Kenyon, S. J., & Luu, J. X. 1999, *AJ*, 118, 1101

Kretke, K. A., & Lin, D. N. C. 2007, *ApJ*, 664, L55

Leinhardt, Z. M., & Stewart, S. T. 2008, *Icarus*, in press (arXiv:0811:0175)

Lin, D. N. C., & Papaloizou, J. 1986, *ApJ*, 309, 846

Lissauer, J. J. 1987, *Icarus*, 69, 249

Masset, F. S., Morbidelli, A., Crida, A., & Ferreira, J. 2006, *ApJ*, 642, 478

Morbidelli, A., Levison, H. F., & Gomes, R. 2008, *The Solar System Beyond Neptune*, 275

Ohtsuki, K., Stewart, G. R., & Ida, S. 2002, *Icarus*, 155, 436

Pollack, J. B., Hubickyj, O., Bodenheimer, P., Lissauer, J. J., Podolak, M., & Greenzweig, Y. 1996, *Icarus*, 124, 62

Rafikov, R. R. 2003, *AJ*, 125, 942

Rafikov, R. R. 2004, *AJ*, 128, 1348

Safronov, V. S. 1969, *Evolution of the Protoplanetary Cloud and Formation of the Earth and Planets*, Nauka, Moscow [Translation 1972, NASA TT F-677]

Spaute, D., Weidenschilling, S. J., Davis, D. R., & Marzari, F. 1991, *Icarus*, 92, 147

Ward, W. R. 1997, *Icarus*, 126, 261

Weidenschilling, S. J. 1977, *MNRAS*, 180, 57

Wetherill, G. W., & Stewart, G. R. 1993, *Icarus*, 106, 190

Wyatt, M. C. 2008, *ARA&A*, 46, 339

Wyatt, M. C., Smith, R., Su, K. Y. L., Rieke, G. H., Greaves, J. S., Beichman, C. A., & Bryden, G. 2007, *ApJ*, 663, 365

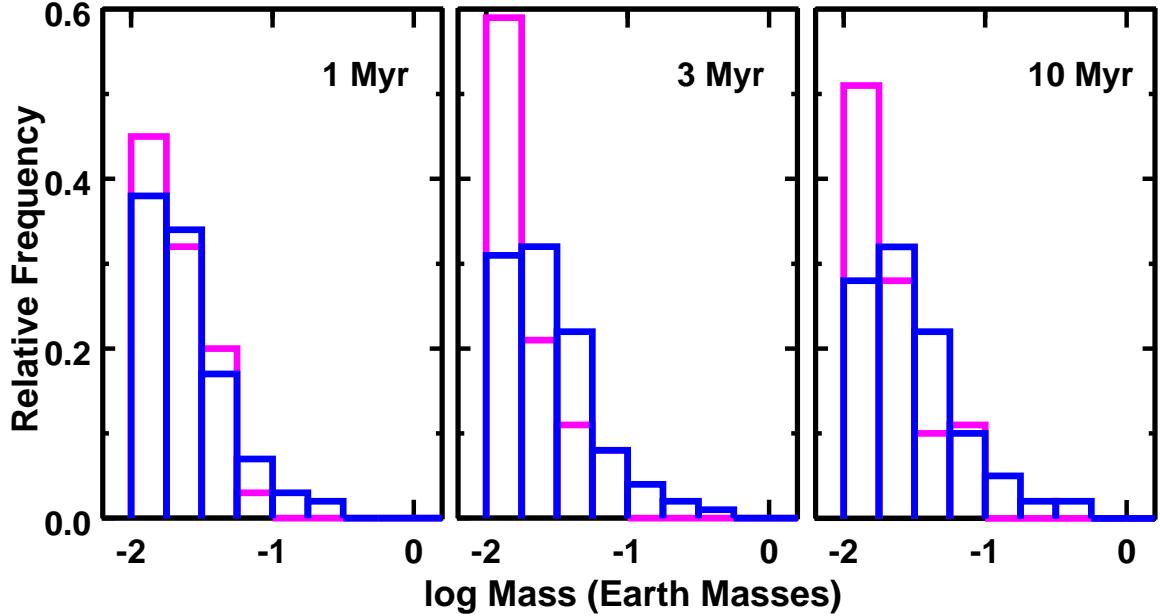


Fig. 1.— Mass histograms at 1 Myr (left panel), 3 Myr (center panel), and 10 Myr (right panel) for coagulation calculations without grain accretion using the  $f_S$  fragmentation parameters at 5 AU. Magenta histograms plot median results for 25 calculations with  $x_m = 1$ ; blue histograms show median results for 25 calculations with  $x_m = 5$ . Independent of disk mass, calculations without grain accretion yield planets with maximum masses  $\lesssim 1 M_\oplus$  in 10 Myr.

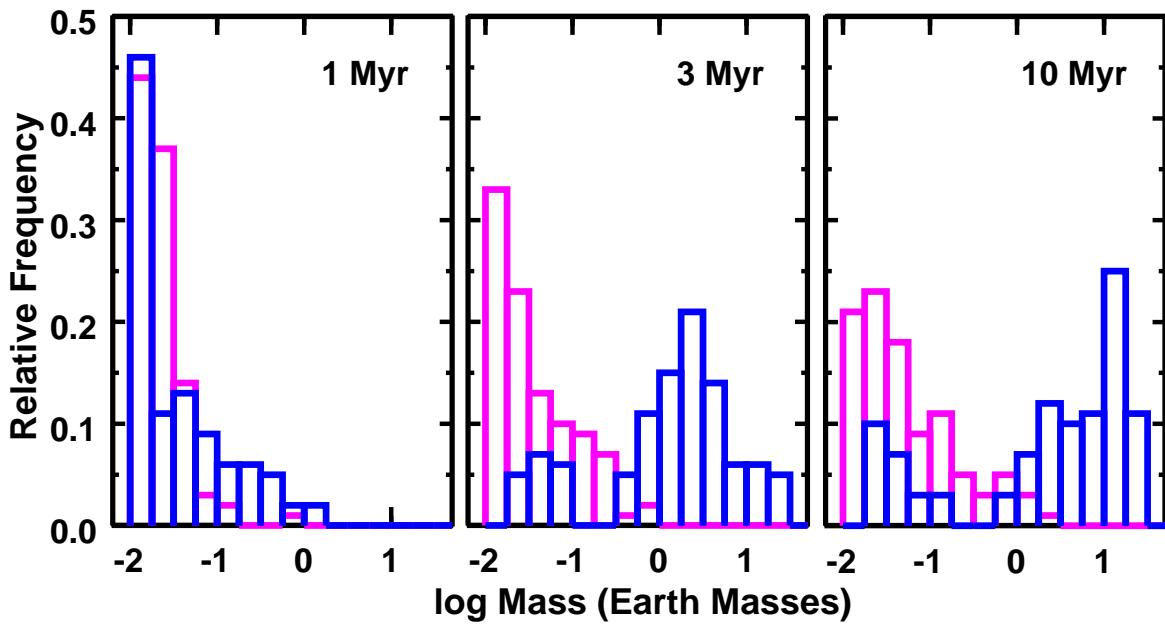


Fig. 2.— As in Fig. 1 for calculations with grain accretion. When large oligarchs can accrete fragments trapped by the gas, disks with  $x_m \gtrsim 1$  produce gas giant cores in 3–10 Myr.